

Quintessence-like Dark Energy in a Lemaître-Tolman-Bondi Metric

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Abstract. We propose to explain the dimming of distant supernovae as the combined effect of dark energy and a Lemaître-Tolman-Bondi (LTB) metric. We take dark energy to have a Quintessence-like Equation Of State $p_{DE} = (w_0 + w_1 \epsilon) \rho_{DE}$ where ϵ is the ratio of LTB shear to LTB expansion.

1. Introduction

The observations that distant supernovae of type Ia appear dimmed have stimulated a vigorous search for models to explain this unexpected fact. Explanations maintaining that the whole Universe is undergoing an accelerated expansion are certainly very bold when one recognizes that SNeIa are seen only out to redshifts $z \approx 1.5$. This locally well-measured part of the Universe is conventionally described by the Friedmann–Lemaître–Robertson–Walker (FLRW) model for a homogeneous, isotropic and unlimited universe, based on Einstein’s equation in four dimensions. The Einstein tensor $G_{\mu\nu}$ encodes the geometry, and the stress-energy tensor $T_{\mu\nu}$ encodes the energy density. Thus modifications to $G_{\mu\nu}$ imply some alternative geometry or metric, whereas modifications to $T_{\mu\nu}$ involve new forms of energy densities called dark energy.

The traditional explanation of the apparent acceleration is the cosmological constant Λ which can be interpreted either as a modification of the geometry or as a vacuum energy term in $T_{\mu\nu}$. This fits observational data well, in fact no competing model fits data better. But the problems with Λ are well known: it must be fine-tuned at some early time to an infinitesimally small value which has no theoretical base, and it offers no explanation to the coincidence problem.

As a cure one has tried time-dependent cosmological constants (Quintessence models), scalar fields acting as dark energy (DE), various modifications of Einstein’s gravity, modified metrics, higher dimensional spacetimes, and spatially inhomogeneous cosmologies. Gravity may even obey field equations other than Einstein’s.

When fitted to observational data no single explanation has been strikingly successful and all models reproduce the Λ CDM model quite well, thereby offering no distinguishable advantage. In this situation we may have to be pessimistic about the simplicity of the laws of Nature and admit that they could be more complicated than expected: we may have to introduce more than one modification at a time. Such many-component models are of course harder to test.

In an attempt of this kind we combined in a previous study [1] the accelerating Chaplygin gas model [2, 3] with the self-decelerating Dvali–Gabadadze–Porrati (DGP) [4, 5] braneworld model. Interestingly, this two-component model did not approach the Λ CDM model in any limit of its parameter space. The DGP deceleration indeed compensated the accelerating effect of the Chaplygin gas to some extent, but not enough so that the result did not satisfactorily fit present observational data.

A way to explain the dimming of distant supernovae without DE may be an inhomogeneous universe described by the Lemaître-Tolman-Bondi solution (LTB) [6, 7, 8] to Einstein’s equation. We could be located near the center of a low-density void which would distort our measurements of the age of the Universe, the CMB acoustic scale, and the Baryon Acoustic Oscillations (BAO). LTB models have both their advocates (e.g. García-Bellido & Haugbølle [9], Dunsby & al. [10]) and their critics (e.g. Zibin & al. [11]). Most of the literature has been cited in the review on observational constraints on inhomogeneous cosmological models without dark energy

by Marra and Notari [12].

In our pessimistic view, future measurements may show that a modified metric is not enough, so that other components may be needed. Some recent steps in this sense was taken by Marra and Paakkonen [13] and Valkenburg [14] who studied a two-component model combining the LTB metric with the cosmological constant, by Marra and Paakkonen [15] who studied an exact spherically-symmetric inhomogeneous model with n perfect fluids, and by J. Lee & al.[16] who coupled a Brans-Dicke scalar field to Horava-Lifshitz gravity.

In the present work we try another two-component model: we combine a Quintessence-like dark energy with the LTB metric. Needless to say, this model has too much freedom to have any predictive power at present, but it does add useful attributes to the LTB model.

2. The model

The LTB model describes general radially symmetric spacetimes in four dimensions. The metric can be described by

$$ds^2 = -dt^2 + X^2(r, t) dr^2 + A^2(r, t) d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, and $A(r, t)$ and $X(r, t)$ are scale functions.

Assuming a spherically symmetric matter source with baryonic+dark matter density ρ_M , dark energy density ρ_{DE} , negligible matter pressure $p_M = 0$ and dark energy pressure density p_{DE} , the stress-energy tensor is

$$T^\mu_\nu = p_{DE} g^\mu_\nu + (\rho_M + \rho_{DE} + p_{DE}) u^\mu u_\nu. \quad (2)$$

Following the notation of ref. [9], the $(0, r)$ components of Einstein's equations, $G^0_r = 0$, imply $X(r, t) = A'(r, t)/\sqrt{1 - k(r)}$, with an arbitrary function $k(r)$ playing the rôle of the spatial curvature parameter. The T^0_0 component of Einstein's equations gives the Friedmann-Lemaître equation in the LTB metric (1)

$$H_T^2 + 2H_T H_L + \frac{k}{A^2} + \frac{k'(r)}{AA'} = 8\pi G (\rho_M + \rho_{DE}). \quad (3)$$

Here $H_T = \dot{A}/A$ is the transversal Hubble expansion, $H_L = \dot{A}'/A'$ the longitudinal Hubble expansion, an overdot designating ∂_t and an apostrophe ∂_r .

The T^r_r components of Einstein's equations give

$$2\dot{H}_T + 3H_T^2 + \frac{k}{A^2} - 8\pi G p_{DE} = 0. \quad (4)$$

Multiplying each term with $A^3 H_T$ this can be integrated over time to give

$$H_T^2 = \frac{F(r)}{A^3} - \frac{k(r)}{A^2} - \frac{8\pi G}{A^3} \int dt A^3 H_T p_{DE}. \quad (5)$$

Here $F(r)$ is an arbitrary time-independent function which arose as an integration constant.

We now assume that dark energy does not interact with matter. The continuity condition $T^{\mu\nu}_{;\nu} = 0$ then gives two separate continuity equations. The one for matter is

$$\dot{\rho}_M(r, t) + [2H_T(r, t) + H_L(r, t)] \rho_M(r, t) = 0, \quad (6)$$

and can be integrated to give

$$\rho_M(r, t) = \frac{F(r)}{A^2 A'} \quad (7)$$

The continuity equation for dark energy is

$$\dot{\rho}_{DE}(r, t) + [2H_T(r, t) + H_L(r, t)] [\rho_{DE}(r, t) + p_{DE}(r, t)] = 0. \quad (8)$$

We now have to specify $p_{DE}(r, t)$ for our model. A very strongly negative pressure like Chaplygin gas is by itself a poor fit to the accelerated expansion. It could be combined with a decelerating LTB geometry, but that would then not describe a void.

Alternatively, Quintessence with an equation of state

$$p_{DE}(r, t) = (w_0 + w_1 a) \rho_{DE}(r, t), \quad (9)$$

is meaningless in LTB, because a scale a valid transversally as well as longitudinally cannot be defined.

We now propose a "Quintessence-like" equation of state

$$p_{DE}(r, t) = (w_0 + w_1 \epsilon) \rho_{DE}(r, t), \quad (10)$$

where ϵ is the ratio of shear to expansion as defined by García-Bellido and Haugbølle [9],

$$\epsilon = \frac{H_T - H_L}{2H_T + H_L}, \quad (11)$$

and w_0, w_1 are constants. In FLRW universes ϵ vanishes identically since $H_T = H_L = H$. The models studied in ref. [9] define a normalized shear such that $\epsilon = 0$ at redshift $z = 0$.

Substituting the equation of state (10) into the continuity equation (8) the latter becomes

$$\frac{\dot{\rho}_{DE}(r, t)}{\rho_{DE}(r, t)} = -(1 + w_0)(2H_T + H_L) - w_1(H_T - H_L). \quad (12)$$

This can be integrated to yield

$$\ln \rho_{DE} = -(2 + 2w_0 + w_1) \ln A - (1 + w_0 - w_1) \ln A' + \ln Q(r), \quad (13)$$

where $\ln Q(r)$ is an integration constant. Thus

$$\rho_{DE} = \frac{Q(r)}{A^{2+2w_0+w_1} A'^{1+w_0-w_1}} = \frac{Q(r)}{(A^2 A')^{1+w_0}} \left(\frac{A'}{A} \right)^{w_1}. \quad (14)$$

Then the dark energy term in Eq. (5) becomes

$$- \frac{8\pi G Q(r)}{A^3} \int dt \frac{\dot{A}}{A^{2w_0+w_1} A'^{1+w_0-w_1}} (w_0 + w_1 \epsilon). \quad (15)$$

3. Discussion

This model now contains three arbitrary functions of r : $k(r), F(r), Q(R)$ and two unknown parameters w_0, w_1 , thus it has very low predictive power. Depending on the parameters w_0, w_1 the pressure can either accelerate or decelerate the expansion, and it can even change from acceleration to deceleration at some critical redshift when $\epsilon = -w_0/w_1$. For $w_1 = 0$ dark energy behaves like Quintessence, and for $w_0 = -1, w_1 = 0$ it becomes the cosmological constant.

Obviously the integral (15) can only be evaluated numerically. This is not motivated to carry out here.

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References

- [1] Roos, M., 2008 *Phys. Lett. B*, 666, 420
- [2] Kamenshchik, A., Moschella, U. & Pasquier, V., 2001 *Phys. Lett. B*, 511, 265
- [3] Bilić, N., Tupper, G. B. & Viollier, R. D., 2002 *preprint* arXiv: astro-ph/0207423
- [4] Dvali, G. R., Gabadadze & Porrati, M., 2000 *Phys. Lett. B*, 485, 208
- [5] Deffayet, D., 2001 *Phys. Lett. B*, 502, 199;
Deffayet, D, Dvali, G. R. & Gabadadze, 2002 *Phys. Rev. D*, 65, 044023
- [6] Lemaître, G., 1933 *Ann. Soc. Sci. Bruxelles*, Ser. 1, 53, 51
- [7] Tolman, R. C., 1934 *Proc. Natl. Acad. Sci. U.S.A.* 20, 169
- [8] Bondi, H., 1947 *Mon. Not. R. Astron. Soc* 107, 410
- [9] García-Bellido, J. & Haugbølle, T., 2008 *J. Cosmol. Astropart. Phys.* 0804, 003
- [10] Dunsby, P., Goheer, N., Osano, B. & Uzan, J.-Ph., 2010 *preprint* arXiv:1002.2397 [astro-ph.CO]
- [11] Zibin, J. P., Moss, A. & Scott, D., 2008 *Phys. Rev. Lett.* 101,251303
- [12] Marra, V. & Notari, A., 2011 *preprint* arXiv: 1102.1015 [astro-ph.CO]
- [13] Marra, V. & Paakkonen, M., 2010, *J. Cosmol. Astropart. Phys.* 12,021
- [14] Valkenburg, W., 2011 *preprint* arXiv:1104.1082 [gr-qc]
- [15] Marra, V. & Paakkonen, M., 2011 *preprint* arXiv:1105.6099
- [16] Lee, J., Lee, T. H. & Oh, Ph., 2011 *Phys. Lett. B*, 701,393